

LECTURE VI

CHAPTER II

Continuity

A continuous function is a function that does not have any abrupt changes in value, also known as discontinuous function.

→ Sufficiently small changes in the input of continuous function result in arbitrarily small changes in its output.

Conditions for Continuity:

(i) $f(x)$ is defined at $x=a$

(ii) $\text{L.H. Limit } f(x) = \text{R.H. Limit } f(x)$
 $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} f(x)$

(iii) $\lim_{x \rightarrow a} f(x) = f(a)$

Q # 01

$$f(x) = |x-3| \quad \text{at } x=3$$

Sol

$$f(x) = |x-3|$$

$$\text{put } x=3$$

$$f(3) = |3-3|$$

$$= 0 \quad \text{--- (1)}$$

R.H. limit

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} |x-3|$$

$$\text{put } x = 3+h$$

$$= \lim_{h \rightarrow 0} |3+h-3|$$

$$= \lim_{h \rightarrow 0} |h|$$

$$= 0 \quad \text{--- (2)}$$

L.H. limit

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} |x-3|$$

$$\text{put } x = 3 - h$$

$$= \lim_{h \rightarrow 0} |3 - h - 3|$$

$$= \lim_{h \rightarrow 0} |-h|$$

$$= \lim_{h \rightarrow 0} h$$

$$= 0$$

Hence,

All conditions are satisfied,
function is continuous.

Ans

Q # 02

$$f(x) = \frac{x^2 - 9}{x - 3} \quad \text{if } x \neq 3$$

$$= 0 \quad \text{if } x = 3$$

Sol

$$f(3) = 0$$

①

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$\therefore a^2 - b^2 = (a - b)(a + b)$$

$$\lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{(x - 3)}$$

$$\lim_{x \rightarrow 3} (x + 3)$$

$$3 + 3$$

$$= 6 \quad \text{---} \quad \textcircled{2}$$

Both conditions not fulfil.
Hence
function is discontinuous.

X

Q #03

$$f(x) = \begin{cases} x-4 & \text{if } -1 < x \leq 2 \\ x^2-6 & \text{if } 2 < x < 5 \end{cases}$$

Sol

at $x=2$

R.H.L

$$\lim_{x \rightarrow 2+0} (x^2 - 6)$$

put $x = 2+h$

$$\lim_{h \rightarrow 0} [(2+h)^2 - 6]$$

$$[(2+0)^2 - 6]$$

$$(4-6)$$

$$-2 \quad \text{---} \quad \textcircled{1}$$

L.H.L

$$\lim_{x \rightarrow 2-0} (x^2 - 6)$$

$$x \rightarrow 2-0$$

put $x = 2-h$

$$\lim_{h \rightarrow 0} [(2-h)^2 - 6]$$

$$[(2-0)^2 - 6]$$

$$(4-6)$$

$$= -2$$

②

3rd Condition

$$f(x) = x - 4$$

$$= 2 - 4$$

$$= -2$$

③

Eq ①, Eq ② & Eq ③

proved that
Function is Continuous.

PRACTICE QUESTIONS

Q# 01

$$f(x) = \begin{cases} x^3 - 27 & \text{if } x \neq 3 \\ \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

at $x = 3$

Q# 02

$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

at $x = 0$

Q# 03

$$f(x) = \begin{cases} \frac{x^2 - a}{a} & \text{if } 0 < x < a \\ 0 & \text{if } x = a \\ a - \frac{a^2}{x} & \text{if } x > a \end{cases}$$

at $x = a$